Fourth Semester B.E. Degree Examination, Dec.2023/Jan.2024 Additional Mathematics – II

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of the matrix by elementary row transformation. $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ (07 Marks)

b. Investigate the consistency of the following equations and if possible find the solutions. 4x - 2y + 6z = 8; x + y - 3z = -1; 15x - 3y + 9z = 21. (07 Marks)

c. Reduce the following matrix to Echelon form and hence find the ranks of A.

$$A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$$

(06 Marks)

OR

2 a. Find all the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

(07 Marks)

b. Apply Gauss elimination method to solve the system of equations, 2x + 5y + 7z = 52, 2x + y - z = 0, x + y + z = 9 (07 Marks)

c. Verify Cayley-Hamilton theorem for the matrix, $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence compute A^{-1} .

(06 Marks)

Module-2

3 a. Solve: $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = 0$.

(07 Marks)

b Solve: $(4D^2 + 4D - 3)y = e^{2x}$

(07 Marks)

c. Solve: $(D^2 + 9)y = \sin 4x$

(06 Marks)

OR

4 a. Solve: $(D^2 + 5D + 4)y = x^2 + 7x + 9$.

(07 Marks)

b. Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + 4y = 4\sec^2(2x)$. (07 Marks)

c. Solve: $(D^2 - 2D + 5)y = 25x^2 + 12$ by the method of undetermined co-efficient. (06 Marks)

Module-3

5 a. Find the Laplace transforms of,

 $e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t$ (07 Marks)

- b. Find the Laplace transform sin2t.cos3t (07 Marks)
 - c. Find the Laplace transform of $(t+1)^2 e^t$. (06 Marks)

OR

6 a. Find the Laplace transform of, $\frac{(1-\cos t)}{t}$. (07 Marks)

b. Find the Laplace transform of,

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$
 if $f(t+2) = f(t)$ (07 Marks)

c. Find the Laplace transform of $t^2.u(t-2)$. (06 Marks)

Module-4

7 a. Find the inverse laplace transforms of, $\frac{4s+15}{16s^2-25}$ (07 Marks)

b. Find the inverse Laplace transform of, $\frac{s+2}{s(s+1)(s+3)}$. (07 Marks)

c. Find the inverse Laplace transform of, $\frac{s+2}{s^2(s+3)}$ (06 Marks)

OR

- 8 a. Find the inverse laplace transform of $log\left(\frac{s+a}{s+b}\right)$. (07 Marks)
 - b. Find the inverse laplace transform of $\frac{1}{s(s+1)^3}$ (07 Marks)
 - c. Solve: $\frac{dy}{dt} + 2y = e^{-3t}$, y(0) = 1. (06 Marks)

Module-5

- 9 a. A bag contains 7 white, 6 red and 5 black balls. Two balls are drawn at random. Find the probability that they will both be white. (07 Marks)
 - b. If A and B are any two events, then prove that $P(A \cup B) = P(A) + P(B) P(A \cap B)$.

 (07 Marks)
 - c. Define the following terms:
 - (i) Trial and Event
 - (ii) Exhaustic Events.
 - (iii) Mutually exclusive events. (06 Marks)

OR

- 10 a. A has 2 shares in a lottery in which there are 3 prizes and 5 blanks; B has 3 shares in a lottery in which there are 4 prizes and 6 blanks. Show that A's chance of success is to B's as 27:35.
 - b. State and prove the Baye's theorem. (07 Marks)
 - c. A bag X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y. (06 Marks)